HYPOTHESIS TESTING AND CONFIDENCE INTERVALS

BEST FOR AFRICA

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OVERVIEW

- Theory of Hypothesis Testing
- Type 1 and Type 2 errors (alpha and beta) and their influence on power
- P-values
- What’s the difference between a Z-statistic and a T-statistic?
- Distinguishing between one-tailed and two-tailed tests
- Confidence Intervals

THEORY OF HYPOTHESIS TESTING

- The underlying theory of hypothesis testing is the basis for which research questions are based
- The null hypothesis, $\mathbf{H_o}$, is the hypothesis of zero difference or no effect
- The alternate hypothesis, $\mathbf{H_1}$ or H_a, is that which is opposite to the null hypothesis
- These hypotheses are created in order to conduct statistical tests simply
- There are only two choices, when you get the p value you decide which one to ‘accept’
- Remember with statistical testing we are not proving anything, we are looking at the result in terms of probability or likelihood of the result we have obtained.

TYPE 1 AND TYPE 2 ERRORS AND POWER

- Type 1 errors are known as false positives, this error is known as alpha and is denoted by the Greek letter: α
- Type 2 errors are know as false negatives, this error is known as Beta and is denoted by the Greek letter: β
- Power can be described as 1- β, and is the probability of reject Ho when in reality it is false
- The factors which influence Type II errors therefore also influence power
- Power increases when sample size increases by reducing standard error, and when the sensitivity of the experimental test allows for small differences to be detected

Reality	**Decision**	
	Fail to reject Ho	Reject Ho
Ho True	Correct Decision	**Type I error** α
Ho False	**Type II error** β	Correct Decision (1-β)

P-VALUES

- P-value is the probability of getting the sample result (or a more extreme result) if the null hypothesis is true
- When we refer to significant level, alpha, this is the probability of making a Type 1 error (false positive)
- Unless a specific reason grants otherwise, it is typical for the researcher to set alpha= 0.05, this gives a 5% chance of producing a false positive
- When the p-value is less than alpha, the probability of getting the sample results if the null hypothesis is true is less than 5%, which is unlikely, and in this case we reject Ho
- When the p-value is more than alpha, the probability of getting the sample results if the null hypothesis is true is more than 5%, and in this case we fail to reject Ho

Z STATISTIC AND T STATISTIC

- For both a simple z test or t test, the test statistic is calculated using the following equation:

$$(\bar{x}-\mu_0) /(s/\sqrt{n})$$

- The resulting critical value is checked using a z table derived from the z distribution, and using a t table from the normalised t distribution respectively
- The limit to using the z statistic in statistical tests, is that it can only be used when the sample size, n >30 because otherwise the z distribution is not normally distributed
- Therefore a t statistic would be required for sample size of 30 and smaller

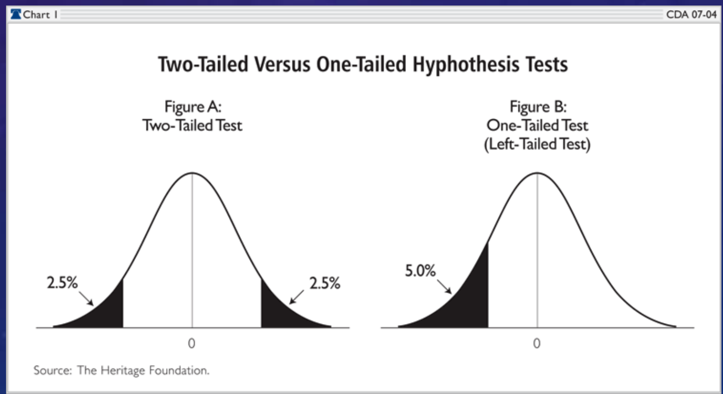
ONE TAILED VS TWO TAILED

- The term “tail” refers to the extremities on a graph, as seen below
- Consider the research question involving two different wheat varieties: does variety X use water more efficiently than variety Y?
- In order to determine if this is a one-tailed or two tailed test, we must construct a null hypothesis and an alternative hypothesis:

Ho: both varieties have the same performance on water use efficiency

H_1: there is a difference in water use efficiency between the two varieties

- Since H1 is only interested in a difference without stating a direction the test is two tailed, therefore it investigates whether the critical value lies within 2.5% of either tail (when a test is two-tailed, the standard alpha=0.05 is divided by 2 and distributed to each tail)
- If, for example, H1 stated variety X is more water efficient than variety Y, this includes a direction and the test would be one tailed.



CONFIDENCE INTERVALS

- Confidence interval is a range in which the true mean lies around the sample mean, at a given confidence level
- For the calculation you take the mean value plus or minus the t value times the standard error of the mean
- Calculation of upper (+) and lower bound (-) of a particular confidence interval :

$$\text{Mean value} \quad \pm \; t \; * \; se$$

- What does a 95% confidence interval mean?

This means that if we repeat the sample 100 times then at 95 times out of a hundred, the true mean lies within a the calculated confidence interval around the sample mean.

- For a given two-sided hypothesis, if the value of the null hypothesis has a 95% confidence interval, the p-value is greater than 0.05
- This relation, shows that at a given α, the corresponding confidence interval is: (1- α)100%

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